ORIGIN OF ATTRACTIVE FORCE.

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The theoretical cycles and epicycles of Ptolemy and his predecessors, the vortices of Descartes, the æther of Newton, were all suggested by an instinctive search for some simple primitive form or cause of motion.

Gravitation is supposed to act under uniform laws in all parts of the universe, and many attempts have been made to refer it to some form of æthereal undulation. Its proportionality, directly to the mass and inversely to the square of the distance, may be readily accounted for on the hypothesis that it is the resultant of infinitesimal impulses, moving with a uniform velocity.

Prof. Stephen Alexander has supposed that the Star System, of which our Sun is a member, is a spiral with several branches. The logarithmic parabola between α Centauri and the Sun, which I have pointed out as controlling the positions of the planets,* confirms this hypothesis, and also furnishes evidence of a material, elastic, slightly compressible æther.

In the spherical undulations of such an æther, propagated like the waves of light, the perimetral disturbance must be π times as great as the synchronous diametral disturbance.

Under the action of central forces, in consequence of the synchronism in all orbits of the same major axis :—

1. A body would describe a circular orbit in the same time that it would oscillate through the centre, over a space equivalent to two diameters. The velocity of the circular oscillation would therefore be $\frac{\pi}{2}$ of the mean velocity of the radial oscillation.

2. A body would oscillate from a circumference to the centre and return, in $(\frac{1}{2})^{\frac{3}{2}}$ of the time of orbital revolution.

3. A body would oscillate through a diameter and return in $(\frac{1}{2})^{\frac{1}{2}}$ of the time of orbital revolution, or in the time which would be required for revolution through the same orbit, with the velocity acquired by infinite impulsion to the circumference.

4. If the velocity of orbital approach to a focus of central force is so retarded, by collisions or otherwise, as to change the orbit from a parabola to a circle, the velocity of the circular oscillation will be $\frac{\pi}{2}$ of the mean velocity of the retarded radial oscillation.

Let us suppose that the planetary groupings, as well as the velocities of planetary revolution, solar and planetary rotation, and solar motion in space, are all resultants of successive infinitesimal impulses, moving with a uniform velocity, and propagated through the medium of a universal æther.

* Proc. A. P. S., Sept. 20, 1872.

If, in consequence of points of inertia, centripetal undulations are established, resulting in a motion of æthereal particles around the centres of inertia, and an accompanying impulsion of denser particles towards the centres, the mean velocity of the circular motion would be one-half as great as that of the originating impulse, and $\frac{\pi}{2}$ as great as the mean velocity of centripetal impulsion.

If a homogeneous rotating globe were aggregated under such centripetal impulsion, the angular orbital velocities of all the particles of the globe would be equally retarded. Rotation is, therefore, merely retarded revolution, and in endeavoring to trace them both to their source, we should compare them at the point of equality.

We know that the hypothetical universal medium is susceptible of undulations, which are propagated with the velocity of light. Therefore let—

V^{λ} = velocity of light, $= 2 \times$ hypothetical mean velocity of æthereal primary rotation, the velocity communicable by the infinitesimal impulses varying between 0 and V^{λ}.

$\frac{V^{\lambda}}{\pi} = \frac{2}{\pi} \times \frac{V^{\lambda}}{2}$ = mean velocity of a perpetual radial oscillation, synchronous with a circular orbital oscillation having a velocity $= \frac{V^{\lambda}}{2}$.

$V' = \frac{{}^{\lambda}}{\pi n}$ = velocity of planetary revolution at the Sun's equator, under the volume due to internal work.

$V'' = \frac{V^{\lambda}}{\pi n^2}$ = velocity of solar equatorial rotation, under the volume due to internal work, = mean velocity of an oscillation through Jupiter's radius vector synchronous with Jupiter's revolution around the Sun; Sun and Jupiter being regarded as constituting a binary Star.

$V''' = 4V''$ = mean velocity of a perpetual radial, or infinitely eccentric oscillation, synchronous with the revolution of the binary Star around its centre of gravity (374335329 seconds) = mean velocity of the binary Star in space.

T', T'' = time of revolution, rotation, for V', V''.

t', t'' = " " " Earth.

τ', τ'' = " " " Jupiter.

$\frac{2V^{\lambda}}{T''}, \frac{2V'}{t''}, \frac{2V'}{\tau''}$ = equatorial g, at Sun, Earth, Jupiter.

$\pi^{\nu}_{\ 3\ 2}$ = ratio of the integral of infinitesimal impulses during revolution in a circular orbit, π^{ν}, to the integral of similar impulses during fall from circumference to centre of same orbit.

$\frac{\pi^6}{32}$ = Neptune's mean heliocentric distance, in units of Earth's mean distance.

$\frac{\pi^5}{32}$ = Saturn's mean distance.

$\frac{\pi^4}{32}$ = Asteroidal mean distance, or twice the mean distance of Mars.

$\frac{\pi^3}{32}$ = Earth's secular mean perihelion distance.

$\frac{\pi^2}{32}$ = Mercury's " " "

$\frac{\pi^{-1}}{32}$ = Major axis of Sun's orbit about centre of gravity of binary Star.

$\frac{\pi^{-2}}{32}$ = Heliocentric distance of linear centre of oscillation of secular mean perihelion centre of gravity of the binary Star.

The ratio of V′ to V″ was determined by supposing Sun's radius to vary from r to n^2r. In such case, $V' \propto \frac{1}{n}$; $V'' \propto \frac{1}{n}{}^2$.

In the following table, A represents the theoretical values of T″ and V″ as estimated from $V\lambda$; B, from Jupiter's distance $\left(V'' = \frac{2\pi\tau'}{\triangle}\right)$; C, the observed values. For T″, C is the mean of the six several estimates by Bianchi and Laugier, Lelambre, Petersen, Spörer, Carrington, and Faye. The Sun's annual motion is given in units of Earth's radius vector, C being Struve's estimate. For V′, A, B, C, are respectively deduced from g at Sun, Earth, Jupiter.

	A	B.	C
T″..................	2203645 sec.	2163907 sec.	2162802 sec.
V′..................	265.66 mi.	261.79 mi.	261.56 mi.
4 V″ × 1 yr.........	1.678 r.v.	1.754 r.v.	1.623 r.v.

The slight discrepancies in these values seem to be attributable to the mean orbital eccentricity of the binary Star, but they are all within the limits of uncertainty of observation. The heliocentric distance of the mean perihelion centre of gravity of the binary Star, is 1.0188 × solar radius; Jupiter's mean orbital eccentricity is .04316.

The correspondence between the theoretical and observed values of the $\frac{\pi^\nu}{32}$ series is given below, in units of Sun's radius. It is specially noticeable that the series groups the principal planets into pairs. The values of the secular mean apsides are taken from Stockwell's "Memoir on the secular variations of the orbits of the eight principal planets."

	Theoretical.	Observed.
Neptune, mean................	6450.776	6453.731
Saturn "	2053.346	2049.514
Asteroid "	653.600	654.760
Earth, perihelion..............	208.048	207.583
Mercury, "	66.224	68.483
Sun, major axis................	2.136	2.132
Primary centre oscillation.......	.679	.679